

SYSTEM FOR REAL TIME TRANSMISSION OF VARIABLE BIT RATE MPEG VIDEO TRAFFIC WITH CONSISTENT QUALITY

This application claims priority from Korean Patent Application
5 No. 2000-067183, filed on November 13, 2000, which is fully
incorporated herein by reference.

BACKGROUND

1. Technical Field:

The present invention relates generally to a video traffic system
10 and, more particularly, to a system for real time transmission of
variable bit rate MPEG video traffic with consistent quality.

2. Description of Related Art:

It is known that variable bit rate (VBR) video provides more
15 constant quality than constant bit rate (CBR) video (see, e.g., "*A
Variable Rate Video Decoder For Synchronous Transfer Mode
Networks*," by W. Verbiest and L. Pinoo, IEEE J. Select. Areas
Commun., Vol. 7, No. 5, pp. 761-770, June. 1989; and "*Packet Video,
Modeling and Signal Processing*", by N. Ohta, Artech House, 1994).
20 The VBR MPEG (Moving Picture Experts Group) coded video traffic is
burst by three different encoding frame types, i.e., I-frame, P-frame,
and B-frame as proposed in "*Generic Coding of Moving Pictures and
Associated Audio: Video*", ISO-IEC/JTC1/SC29/WG11/ NO801rev,

MPEG Committee Draft: Rec. H. 262, April, 1995. The VBR video is highly time-variant due to successive scenes having different contents as proposed in "*Bandwidth Renegotiation For VBR Video Over ATM Networks*," by D. J. Reininger, D. Raychaudhuri and J. Y. Hui, IEEE J. Select. Areas Commun., Vol. 4, No. 6, pp. 1076-1086, August, 1996.

A video with constant quality can be obtained by transmitting an unconstrained VBR providing similar quantization parameter to all frames, such as proposed in the above reference by D. J. Reininger, et al., as well as "*MPEG Coding For Variable Bit Rate Video Transmission*", by P. Pancha and M. El Zarki, IEEE Commun. Mag., Vol. 32, No. 4, pp. 54-66, May. 1994. Since an ATM (Asynchronous Transfer Mode) network provides limited bandwidth, however, the ATM network requires a constrained VBR specified by three parameters of peak rate, sustainable rate, and maximum burst length. Smoothing (or shaping) is one of the most dependable methods that efficiently improves network utilization and achieves constrained VBR transmission at an acceptable delay cost, as proposed in "*Performance Models of Statistical Multiplexing In Packet Video Communications*", by B. Maglaris, IEEE Trans. Commun., Vol. 36, No. 7, pp. 834-844, July. 1998, as well as "*Analysis of ON-OFF Source Shaping For ATM Multiplexing*", by F. Bernabei, Proc. INFOCOM, pp. 1330-1336, 1993.

In addition to smoothing, bandwidth renegotiation is another important system for the transmission. However, undesirable operation

of the smoothing and bandwidth renegotiation may give rise to unexpected encoder buffer overflow, decoder buffer overflow, or violation of the renegotiated bandwidth, which causes unwanted loss of data. The undesirable loss of data seriously deteriorates the

5 transmitted video quality. Therefore, a method for controlling a joint encoder and channel rate is needed which considers an allocated bandwidth and sizes of the encoder and decoder buffers, such as proposed by "*Constraints on Variable Bit Rate Video for ATM Networks*", A. R. Reibman and B. G. Haskell, IEEE Trans. Circuit Syst. Video Technol., Vol. 2, No. 4, pp. 361-372, December. 1992; and "*Joint Encoder and Channel Rate control of VBR Video over ATM Networks*", 10 W. Ding, IEEE Trans. Circuit Syst. Video Technol., Vol. 7, No. 2, pp. 266-278, April, 1997.

Reininger et al. proposes method for bandwidth renegotiation 15 without traffic smoothing, and an method for controlling encoder rate corresponding to negotiated traffic parameters. Reibman, et al. discloses that constraints imposed on the encoded video bit rate (i.e., encoded video transmission rate) are caused by encoder and decoder buffering. Ding proposes a method for traffic smoothing, joint encoder, 20 and channel rate control, wherein the negotiated peak rate is very large while the employed peak rate is very small. That is, the allocated peak rate is 4 times higher than the rate to have been maintained (about 180 kbits/frame) if the used peak rate is only half of the allocated peak rate.

In the methods disclosed by Reibman, et al. and Ding, traffic parameters for bandwidth renegotiation and rate control cannot be obtained by on-line processing. Some techniques for traffic smoothing for VBR MPEG video are disclosed in the following references:

- 5 “Source Traffic Smoothing and ATM Network Interfaces for VBR MPG Video Encoders”, by K. Joseph and D. Reininger, Proc. ICIP, pp. 1761-1767, 1995); “Adaptive Prediction and Smoothing of MPEG Video in ATM networks”, by S. Jung and J. S. Meditch, Proc, ICC, pp. 832-836, 1995); and “An Algorithm for Lossless Smoothing of MPEG Video” by S.
- 10 S. Lam, S. Chow and D. K. Y. Yau, ACM SIGCOMM, pp. 281-293, 1994. These techniques, however, have not considered bandwidth renegotiation.

SUMMARY OF THE INVENTION

The object of the present invention is to provide an efficient

15 system and method for real-time transmission of VBR MPEG video traffic with consistent quality.

According to one aspect of the invention, a variable bit rate video system comprises a video encoder for encoding video data, an encoder buffer for storing data encoded from the video encoder, an

20 encoder rate controller for estimating the number of bits quantized from a target quantization parameter during a frame of the video data corresponding to discrete cosine transform (DCT) coefficients of the frame, and for providing quantization parameters based on slice level to

the encoder, a channel rate controller for generating parameters for smoothing and bandwidth renegotiation corresponding to the number of bits generated from the encoder rate controller, a network for generating negotiated parameters corresponding to the parameters generated from the channel rate controller, and a Leaky Bucket counter for transmitting the video data stored the encoder buffer through the network.

In a system for transmission of variable bit rate MPEG video traffic with consistent quality, each frame is encoded with quantization parameter generated from an encoder rate controller, and an encoded data is transmitted to the encoder buffer. Transmission rate about each frame interval is determined by the channel rate controller at the beginning of the frame interval. Data transmission buffered at the encoder buffer is first regulated by the Leaky Bucket counter, and the regulated data is transferred to a decoder buffer in a video receiving system through network.

In another aspect of the invention, a method for transmission of variable bit rate MPEG video traffic with consistent quality, comprises the steps of: encoding frames of video data and generating encoding information associated therewith; estimating a number of bits quantized from a target quantization parameter during a given frame of said video data based on encoding information of the given frame; estimating a maximum number of bits capable of being generated based on network traffic parameters and buffer sizes; computing a target bit rate using the

estimated number of bits quantized with the target quantization parameters and the estimated maximum number of bits; generating quantization parameters to control the actual encoding rate, if necessary, so that a number of actual bits generated does not exceed the target bit rate; and dynamically negotiating with a network to generate traffic parameters for smoothed traffic and required bandwidth.

These and other aspects, features, and advantages of the present invention will become apparent from the following detailed description of the preferred embodiments, which is to be read in connection with the accompanying drawings.

BRIEF DESCRIPTION OF THE DRAWING

The Figure is a block/flow diagram illustrating a system/method for transmitting and receiving VBR video according an embodiment of the present invention.

DESCRIPTION OF PREFERRED EMBODIMENTS

In one embodiment of the invention, a system and method are provided that employs both a joint encoder and channel rate control protocol that complies with negotiated traffic parameters and constraints imposed by encoder and decoder buffers. To maintain a consistent visual quality, a target quantization parameter (target Q) that is applied to each frame is selected based on factors such as encoder and decoder buffer sizes and delay. Low target Q becomes more frequently applied by encoder rate control, and resulting in significant

fluctuations of visual quality of transmitted video.

For a bit rate control method, the present invention affords an efficient method for selecting target Q and dynamic bandwidth renegotiation for smoothed traffic. In a preferred bandwidth renegotiation method, the traffic parameters for the smoothed traffic and required bandwidth are estimated in real-time, and dynamically renegotiated with networks. A preferred traffic smoothing method comprises analyzing the conditions of the traffic parameters for transmission of VBR MPRG video without delay, in which peak rate can be smoothed to the sustainable rate. A coding result of unconstrained VBR or CBR with same average bandwidth are compared with the proposed algorithm using simulations. Simulation results of the present invention have shown that a consistent quality video can be transmitted in accordance with the preferred algorithm.

It is assumed that the consistent visual quality of transmitted video can be achieved when similar target quantization parameters are applied to the same type of frames within a scene. A preferred system and method for transmitting/receiving VBR MPEG video will now be described in further detail.

The Figure is a block diagram illustrating a system for transmitting and receiving VBR video data in accordance with an embodiment of the present invention. It is to be understood that the Figure further depicts a method for transmitting and receiving VBR

video data according to one aspect of the present invention. The system comprises a VBR video transmission system 100 and a VBR video receiving system 300. The transmission system 100 comprises a video encoder 110, an encoder buffer 120, a Leaky Bucket (LB) counter 150, an encoder rate controller (ERC) 130, and a channel rate controller (CRC) 140. The VBR video receiving system 300 comprises a decoder buffer 310 and a video decoder 320. The ERC 130 and CRC 140 control the rate generated from video encoder 110 and the rate transmitted from a network 200, respectively, satisfying the imposed constraints.

Each frame is coded with a quantization parameter $E_i(Q_{igt})$ provided by the encoder rate controller 130. Coded data from the video encoder 110 is transferred to the encoder buffer 120. Transmission rate for each frame interval is determined by the channel rate controller 140 at the beginning of the frame interval. The transmission of buffered data is first regulated by Leaky Bucket (LB) counter 150, and regulated data is transferred to the decoder buffer 310 via network 200. It is assumed that a delay within the network is constant and, therefore, the delay within the network can be ignored for purposes of designing rate control protocol according to the invention.

In accordance with a preferred embodiment of the invention, the system selects an appropriate set of target quantization parameters for each frame type. The joint encoder and channel rate control comply

with the constraints imposed by the encoder and decoder buffer sizes and the negotiated traffic parameters.

The following notations are used herein for the description. $Q_{tgt} = Q_I, Q_P, Q_B$ comprises a set of target quantization parameters, where Q_I , Q_P , and Q_B are the parameters associated with the I-frame, P-frame, and B-frame, respectively. T denotes time duration corresponding to an uncoded frame. The terms E_i and R_i respectively represent the number of bits generated and transmitted during the interval $[(i-1)T, iT]$. E_i^+ and R_i^+ denote the maximum number of bits capable of being generated and transmitted during i -th frame period, respectively. E_i^{tgt} denotes the target number of bits to be generated during i -th frame period. $E_i(Q_{tgt})$ and $\tilde{E}_i(Q_{tgt})$ denote the actual and estimated number of bits as being quantized with target quantization parameters Q_{tgt} during i -th frame, respectively. D represents delay within frames, that is, after the decoder begins to receive data, it waits DT seconds before starting to decode. B_i^e and B_i^d denote the buffer states in accordance with bits of encoder and decoder at the end of the i -th frame period $t=iT$, respectively. B_{max}^e and B_{max}^d denote the buffer sizes in bits of encoder and decoder, respectively. L_{max} denotes the size of the Leaky Bucket (LB) counter in bits. L_i is the state of LB counter in bits at the end of the i -th frame period $t=iT$.

The operations of the video transmission system 100 are as follows. It is assumed that the i -th frame is available for being coded.

The encoder rate controller (ERC) 130 estimates $\tilde{E}_i(Q_{gt})$ when the frame is encoded with target quantization parameters Q_{tgt} by using information obtained from the current frame such as discrete cosine transform (DCT) parameter as well as previously coded frame. The ERC 130
 5 calculates the target bit rate E_{tgt} by using $\tilde{E}_i(Q_{gt})$ and E_i^+ provided from the channel rate controller (CRC) 140. Then, the ERC 130 controls the actual encoder rate (which preferably does not exceed the target rate) by appropriately assigning quantization parameter on the basis of slice-level, i.e. $q_{i,k}$, ($k = 0, 1, \dots, N_{slice} - 1$).

10 The CRC 140 estimates traffic parameters for smoothing and bandwidth renegotiation by using $E_j(Q_{tgt})$, [$j=i-G_{N+1}, \dots, I$] that is provided from the ERC 130. If necessary, the CRC 140 renegotiates with networks for the newly required bandwidth. And the CRC 140 determines the actual rate transmitted from encoder buffer 120 by using
 15 the calculated upper bounds E_i^+ and R_i^+ satisfying the constraints imposed by encoder and decoder buffers and the negotiated parameters. If necessary, bits are filled in order to avoid underflow within the encoder buffer 120.

The Leaky Bucket (LB) counter 150 is a device for policing
 20 traffic, which increases R_i bits and decrease R_s bits during the i -th frame coding period. When the LB counter is full, predetermined data transferred from the encoder buffer are discarded. For example, if $R_i = R_s$ (or $K_p = 0$), the LB counter size $L_{max} = 0$, as the result, the channel

rate is limited to R_s .

The video decoder 320 starts decoding i -th frame at $t=(i+D-1)T$, and completes the decoding at $t=(i+D)T$. Therefore, all data of the i -th frame should be transmitted to the decoder buffer 310 by $t=(i+D)T$.

5 The constraints on encoder and channel rate are described in detail in the above-identified references by Reibman and Haskell. A preferred range of encoder and channel rates imposed by traffic parameters of buffer sizes of the encoder and decoder is as follows.

The video decoder 320 waits during D frame periods to start
10 decoding after data begins to be received. The encoder rate E_i should be transmitted before $t=(i+D)T$. When the channel rate is R_i , the buffer states are described as equations (1) and (2).

$$B_i^e = B_{i-1}^e + E_i - R_i, \quad 0 \leq B_i^e \leq B_{\max}^e \quad (1)$$

$$B_i^d = B_{i-1}^d + R_i - E_{i-D}, \quad 0 \leq B_i^d \leq B_{\max}^d \quad (2)$$

15 where $B_i^e = B_i^d = 0$ and $E_i = R_i = 0$ for $i \leq 0$.

The traffic policer comprises an LB counter 150 and detects a state of the virtual buffer inside the network 200. The LB counter receives R_i bits and outputs R_s during the i -th frame period. When the LB counter size is L_{\max} , the LB counter state at the end of the i -th frame
20 period $t=iT$ is

$$L_i = \min\{L_{\max}, \max\{0, L_{i-1} + R_i - R_s\}\} \quad (3)$$

where $i=1,2,\dots$ and $L_0=0$.

The system cannot store more than $B_{\max}^e + B_{\max}^d$ bits at any given

time. If the number of bits coded from successive D frames is larger than the total size of system buffers $B_{\max}^e + B_{\max}^d$, some coded data may be lost by the buffer overflow. Therefore, the following relationship is valid for $i \geq D$.

$$\sum_{j=0}^{D-1} E_{i-j} \leq B_{\max}^e + B_{\max}^d \quad (4)$$

The above relationship is readily obtained from Equations (1) and (2). It is noted that the encoder rate (or the target quantization parameter) and the delay are closely related to the encoder buffer 120 and decoder buffer 310 sizes.

If the channel rate is selected within the maximum rate allowed by the decoder buffer 310 and LB policer, the delay constraints can be guaranteed. From equations (2) and (3), the maximum number of bits R_i^+ capable of being transmitted during the i-th frame period can be represented as follow.

$$R_i^+ = \min\{B_{\max}^d - B_{i-1}^d + E_{i-D}, L_{\max} - L_{i-1} + R_s\} \quad (5)$$

As mentioned above, the encoder rate E_i preferably satisfies the conditions of the decoder buffer overflow at decoding time as well as the encoder buffer overflow at encoding time. Therefore, the maximum number of bits E_i^+ capable of being generated by video encoder can be obtained by equations (1) and (2) as follows.

$$E_i^+ = \min\{B_{\max}^e - B_{i-1}^e + R_i^+, B_{i+D-1}^d + R_{i+D}^+\} \quad (6)$$

where B_{i+D-1}^d and R_{i+D}^+ can be calculated by using E_j , $j=i, i+1, \dots$,

i+D-1 and equation (5), as described in the above-identified reference by Reibman. Additionally, the actual encoder rate E_i and the actual channel rate R_i preferably satisfy the encoder underflow condition, $B_{i-1}^e \leq R_i - E_i$. However, when the encoder buffer underflow is expected,
 5 it can be avoided by stuffing bits appropriately.

If the number of bits coded from successive D frames are larger than the system buffer size, $B_{\max}^e + B_{\max}^d$, some frames following them should be coded with larger quantization parameters than the set of target quantization parameter Q_{tgt} so as to avoid loss of coded data.

10 This causes degradation of visual quality of transmitted video, and as a result, an unintentional fluctuation of the quality of video may occur. The fluctuation can be avoid by the right selection of Q_{tgt} considering the system buffer size.

It is assumed that D is less than or equal to the GOP (group of picture) size. When one of D frames is an I-frame, the largest buffer is
 15 preferred. The D frames comprise one I-frame, N_P P-frames, and N_B B-frames. If the number of bits of the I-frame, P-frame, and B-frame are ratios of x:y:z, respectively, all frames will have a similar visual quality, and the number of bits capable of being allocated to I-frame is
 20 given as follows.

$$E_I = \frac{x(B_{\max}^e + B_{\max}^d)}{x + yN_P + zN_B} \quad (7)$$

It is noted that x, y, and z are empirically selected, such as

x:y:z=4:2:1. In the reference “*An Accurate Control of MPEG Video By Rate-Codewords Modeling*,” by T. Y. Kim and J. K. Kim, Proc.

ISCAS’97, pp.1261-1264, 1997, a bit rate estimation model

(transmission rate) and an actual bit rate control algorithm (actual

- 5 transmission rate control algorithm) are based on the linear relationship between codeword count and real bit count, and the target quantization parameter for I-frame is given as follow.

$$Q_i^* = \arg \min_{Q_i} |C_i(Q_i) - \tilde{C}_i(E_i)| \quad (8)$$

- where $C_i(Q_i)$ means the number of codewords generated by using
 10 the I-frame quantization parameter Q_i calculated from the pre-analysis of discrete cosine transform (DCT) coefficients, and $\tilde{C}_i(E_i)$ is the number of codewords estimated from the target bit count E_i given by equation (7). That is,

$$\tilde{C}_i(E_i) = \frac{E_i - \beta}{\alpha} \quad (9)$$

- 15 where α and β are model parameters calculated from the coded results of previous frame with the same type. The P-frame and B-frame quantization parameters Q_P and Q_B are determined, so that the peak signal to noise ratio (PSNR) of P-frame and B-frame may be equivalent to I-frame. According to the present invention, $\{Q_I, Q_P,$
 20 $Q_B\} = \{3, 4, 5\}$ for $Q_I \leq 3$, $\{Q_I, Q_P, Q_B\} = \{Q_I, Q_I+1, Q_I+2\}$ for $4 \leq Q_I \leq 9$, and $\{Q_I, Q_P, Q_B\} = \{Q_I, Q_I+2, Q_I+4\}$ for $10 \leq Q_I$.

It is noted that the set of target quantization parameter Q_{tgt} should be selected before each frame is coded. Indeed, whenever there are scene changes, modification of the quantization parameters using the above procedure would be very efficient for keeping consistent visual quality. Furthermore, even when there is no scene change, if many frames are coded by quantization parameters different from the Q_{tgt} , the Q_{tgt} should be modified.

It is assumed that $E_i(Q_{tgt})$ and $\tilde{E}_i(Q_{tgt})$ are the actual and estimated number of bits quantized with Q_{tgt} from the i -th frame, respectively. In case of test model 5 (TM5), $\tilde{E}_i(Q_{tgt}) = X / Q_{tgt}$, where X is global complexity of the previous frame with the same type. In the above reference by Kim et al., the number of bits $\tilde{E}_i(Q_{tgt})$ is estimated exactly by using the linear relationship between the number of codewords and the estimated number of bits as follows.

$$\tilde{E}_i(Q_{tgt}) = \alpha C_i(Q_{tgt}) + \beta \quad (10)$$

where α and β are same as the parameters of equation (9). In case of ideal estimation, $\tilde{E}_i(Q_{tgt})$ is equal to $E_i(Q_{tgt})$. The method by the linear relationship provides much more excellent estimation performances than the TM5 proposed by the paper of Kim et al. In a preferred embodiment of the present invention, the bit rate estimation model disclosed by Kim et al is extended for the encoder rate control method.

If the i -th frame is coded with the Q_{tgt} , the number of bits $\tilde{E}_i(Q_{tgt})$

is generated and the encoder is controlled so as to be $E_i \leq E_i^+$. As a result, if $\tilde{E}_i(Q_{igt}) \leq E_i^+$, a target bit rate E_i^{tgt} becomes $\tilde{E}_i(Q_{igt})$, and if $\tilde{E}_i(Q_{igt}) > E_i^+$, E_i^{tgt} becomes E_i^+ . Then the encoder is controlled so that $E_i \leq E_i^{tgt}$. However, because of the inaccuracy of encoder rate control
 5 generated from inaccurate bit rate estimation, the target bit rate E_i^{tgt} is preferably estimated as follows.

If $\tilde{E}_i(Q_{igt}) < \alpha E_i^+$, $E_i^{tgt} = \tilde{E}_i(Q_{igt})$ and quantize with Q_{igt} (11)

and if $\tilde{E}_i(Q_{igt}) \geq \alpha E_i^+$, $E_i^{tgt} = \alpha E_i^+$ and control the encoder rate

where the constant α is selected so that the actual bit rate does
 10 not exceed the bit rate bound, i.e. $E_i \leq E_i^+$, and in the present invention, it is preferred that 0.9 is used for the constant α . It is preferred that, for VBR MPEG video, all frames are coded with the target parameter Q_{igt} to maintain consistent visual quality from frame to frame.

However, because of several constraints imposed by traffic parameters
 15 and buffer sizes, the actually generated bit counts E_i may be different from the real bit counts $E_i(Q_{igt})$ coded by using the Q_{igt} . If the encoder rate is controlled so that $E_i \leq E_i^+$, then overflows within the encoder buffer can not occur. Underflows that may be generated at $E_i \ll E_i^+$ can be solved by stuffing bits.

20 However, if $E_i > E_i^+$, some data may be lost by overflows in the encoder buffer or underflows in the decoder buffer. This is preferably solved by appropriately selecting quantization parameters of each slice in a frame as follows.

If $q_{i,k}$ is the actually applied quantization parameter to k -th slice in the i -th frame, $k=0, 1, \dots, N_{\text{slice}}-1$ and the N_{slice} means the number of slice in a frame. The $q_{i,k}$ comprises $\hat{q}_{i,k}$ and $\Delta q_{i,k}$, where $\hat{q}_{i,k}$ is an initial parameter determined by E_i^{tgt} and $\Delta q_{i,k}$ is an additional
 5 parameter selected by considering the bit counts generated until $(k-1)$ th slice.

First, it is required to determine the value of $\hat{q}_{i,k}$ for slice k . If $E_i^{\text{tgt}} = \tilde{E}_i(Q_{\text{tgt}})$, $\hat{q}_{i,k}$ sets the target quantization parameter Q_{tgt} , and if $E_i^{\text{tgt}} = \alpha E_i^+$, $\hat{q}_{i,k}$ is properly selected so as to meet the target bit rate E_i^{tgt}
 10 by using the method proposed in the paper of Kim et al. The above two cases are derived from equation (11).

After determining the value of $\hat{q}_{i,k}$ for slice k , $\Delta q_{i,k}$ is determined. The $\Delta q_{i,k}$ is selected by considering the actually generated bit counts and the upper bound of bit counts to $(k-1)$ slice in
 15 the i -th frame. If $E_{i,k}^{\text{tgt}}$ is the maximum number of bits for slice k in the i -th, $\Delta q_{i,k}$ can be obtained as follows.

$$\Delta q_{i,k} = \min\{31, \Delta q_{i,k-1} + 1\}, \text{ if } \left(\sum_{j=0}^{k-1} E_{i,j} > \sum_{j=0}^{k-1} E_{i,j}^+ \right) \quad (12)$$

$$\Delta q_{i,k} = \max\{0, \Delta q_{i,k-1} - 1\}, \text{ if } \left(\sum_{j=0}^{k-1} E_{i,j} \leq \sum_{j=0}^{k-1} E_{i,j}^+ \right)$$

where $\Delta q_{i,0} = 0$, $E_{i,j}$ is the actually generated number of bits from slice j
 20 in the i -th frame. Finally, the actual quantization parameter $q_{i,k}$ is given by

$$q_{i,k} = \max\{1, \min\{31, \hat{q}_{i,k} + \Delta q_{i,k}\}\} \quad (13)$$

where the value of $q_{i,k}$ is preferably limited to between 1 and 31 because $q_{i,k}$ is preferably used as a quantization step size in TM5.

The number of bits capable of being transmitted will be limited to
 5 $R_i \leq B_{i-1}^e + E_i$ from the condition of encoder underflows. Because the actual encoder rate E_i cannot be known at the starting time of encoding of the i -th frame, the channel rate is redefined by the target number of bits E_i^{tgt} of equation (11) as follows.

$$R_i = \min\{B_{i-1}^e + E_i^{tgt}, B_{\max}^d - B_{i-1}^d + E_{i-D}, L_{\max} - L_{i-1} + R_s\} \quad (14)$$

10 The encoder rate is controlled so that $E_i = E_i^{tgt}$. As a result, the encoder buffer underflows that may occur as $R_i = B_{i-1}^e + E_i^{tgt}$ can be solved by stuffing bits.

The channel rate R_i that is computed as indicated above is the maximum rate allowed by the decoder buffer and LB counter. This
 15 method advantageously causes the encoder buffer 120 maintain large empty area as described in equation (6) of $B_{\max}^e - B_{i-1}^e$. The empty area enlarges the upper bounds of encoder rate. If the encoder buffer becomes full (B_{i-1}^e is nearly equal to B_{\max}^e), E_i^+ is limited to R_i^+ .
 Therefore, control of the encoder rate is less frequently applied by the
 20 larger upper bounds of encoder rate, resulting in more consistent quality video.

Since asynchronous transfer mode (ATM) network provides a bandwidth constrained by a limit of available bandwidth or traffic

contracts with each connection, it is inevitable for the connection to transmit constrained VBR. For the control of the constrained VBR, ATM forum specifies parameters such as peak rate R_p , sustainable rate R_s , and maximum burst length MBL.

5 The maximum bandwidth for transmitting data that is coded with Q_{tgt} to guarantee the transmission of VBR MPEG video closely related to visual quality needs to be reserved within the network 200. It can be achieved by reserving the bandwidth as a peak rate of the video traffic. However, since a video sequence consists of each scene
10 together with very different characteristics of generating bits, low network utilization is inevitable.

 The smoothing (or shaping) is one of the most promising methods to reduce the peak rate of the traffic, and efficiently improve network utilization. However, if the smoothing process does not take into
15 consideration various probable characteristics of the traffic, the problem of low network utilization still remains. The bandwidth renegotiation is another important method for overcoming the problem. The present invention provides methods for smoothing and bandwidth renegotiation.

20 Since frame types periodically appear based on a pre-defined group of pattern (GOP) of a MPEG video, traffic pattern generated from the MPEG video source is basically determined at the GOP level. Thus, to make a decision of renegotiation, it is reasonable to select an interval

for monitoring the traffic during a multiple of GOP size. Preferably, one GOP period comprising G_N frames is utilized for monitoring the traffic.

$E_{avg}(Q_{tgt})$ and $E_{max}(Q_{tgt})$ denote the average number and maximum number of bits per frame of GOP period including the current frame i , respectively. The average number and maximum number of bits per frame after the i -th frame are preferably coded as follows.

$$E_{avg}(Q_{tgt}) = \frac{1}{G_N} \sum_{j=0}^{G_N-1} E_{i-j}(Q_{tgt}) \quad (15)$$

$$E_{max}(Q_{tgt}) = K_m \times \max\{E_i(Q_{tgt}), E_{i-1}(Q_{tgt}), \dots, E_{i-G_N+1}(Q_{tgt})\}$$

where $K_m(\geq 1)$ is a constant, and the purpose of the constant K_m is to offer a margin for selecting the maximum bit rate $E_{max}(Q_{tgt})$. For example, if $K_m = 1$, $E_{max}(Q_{tgt})$ is same as the maximum bit rate during the GOP period, and if $K_m = 2$, $E_{max}(Q_{tgt})$ becomes twice of the maximum bit rate of the GOP period. The generated number of bits $E_j(Q_{tgt})$ ($j=i-G_N+1, \dots, i$) in equation (15) is determined as $E_j(Q_{tgt})=E_j$ (for $i-G_N+1 \leq j < i$) if the j -th frame was coded with Q_{tgt} , otherwise $E_j(Q_{tgt}) = \tilde{E}_j(Q_{tgt})$. $E_j(Q_{tgt}) = \tilde{E}_j(Q_{tgt})$ is defined as an absolute value because the i -th frame was not coded.

\tilde{R}_s and \tilde{R}_p denote the normal sustainable rate and peak rate for the passed GOP period including the current frame i , respectively. \tilde{R}_s and \tilde{R}_p are preferably given by:

$$\tilde{R}_s = E_{avg}(Q_{tgt}) + K_s \{E_{max}(Q_{tgt}) - E_{avg}(Q_{tgt})\} \quad (16)$$

$$\tilde{R}_p = \tilde{R}_s + K_p \{E_{\max}(Q_{igt}) - \tilde{R}_s\}, \quad (17)$$

where K_s and K_p are constants satisfying the conditions of $0 \leq K_s \leq 1$ and $0 \leq K_p \leq 1$, respectively. Thus, \tilde{R}_s and \tilde{R}_p satisfy the conditions of $E_{avg}(Q_{igt}) \leq \tilde{R}_s \leq E_{\max}(Q_{igt})$ and $\tilde{R}_s \leq \tilde{R}_p \leq E_{\max}(Q_{igt})$, respectively. If $K_p=1$, the traffic is not smoothed, i.e. $\tilde{R}_p = E_{\max}(Q_{igt})$, and defined as ‘no smoothed VBR’. While if $K_p < 1$, the maximum rate $E_{\max}(Q_{igt})$ is smoothed to $\tilde{R}_p (< E_{\max}(Q_{igt}))$. In this case, smoothing buffers are preferably implemented.

The object of bandwidth renegotiation is to dynamically allocate a specific bandwidth that varies based on scene content. A preferred index of traffic variation trend UPC_{var} ($=0, 1, 2, \dots$) that is given by comparing the normal sustainable rate \tilde{R}_s together with the current sustainable rate R_s is as follows.

If $(\tilde{R}_s < (1-b)R_s)$, decrement UPC_{var} (18)
 else if $((1+b)R_s < \tilde{R}_s)$, increment UPC_{var}
 else if $(0 < UPC_{var})$, decrement UPC_{var}
 else if $(UPC_{var} < 0)$, increment UPC_{var}
 else $UPC_{var} = 0$

where $0 \leq b < 1$ and UPC_{var} is initially set to zero. Whenever the bandwidth will be renegotiated, or not renegotiated, the bandwidth is preferably determined by comparing UPC_{var} and a predefined threshold value X (≥ 0). That is,

$$\text{if } |UPC_{var}| > X,$$

bandwidth renegotiation by $\{R_s, R_p, MBL\} = \{\tilde{R}_s, \tilde{R}_p, \tilde{R}_p\}$ (19)

It is noted that the values of b and X determine a rate of bandwidth renegotiation. If the value of b is close to 1 or X is set to a large value, the bandwidth renegotiation hardly occurs. The time of bandwidth renegotiation T_{nego} is the frame periods consuming between the beginning time and the end time of the bandwidth renegotiation. It is assumed that the bandwidth renegotiation is carried out from the next I-frame until before T_{nego} . It means that the current traffic parameters are replaced by new traffic parameters at the time of the next I-frame.

10 In simulations, if $D \leq 3$ frames, $T_{nego} = 3T$, otherwise $T_{nego} = DT$.

The LB counter size based on the negotiated traffic parameters is given by

$$L_{max} = R_p - R_s \quad (20)$$

It means that the maximum burst length MBL in peak rate equals to the negotiated bit rate R_p , i.e. $MBL = R_p$ when $L_i = 0$. L_{max} is an adequate size of LB counter in order that the LB counter is increased by $R_s = R_p$ bits and decreased by R_s bits during frame period i .

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When a scene change occurs from simple to complicated scene, the current traffic parameters may be affected even though a changed frame is quantized with a large quantization parameter. This is because the previous traffic parameters are used for the changed frame that has different characteristics from the previous scene. To solve the problem, a lower bound of sustainable rate R_s is preferably defined.

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VBR traffic may be transmitted without delay ('no delay VBR'), or delayed in some parts by a smoothing mechanism ('smoothed VBR').

First, in the case of 'no delay VBR', ideal transmission of VBR traffic requires no delay, i.e. no buffer, and the 'no delay VBR' can be achieved when a limitless bandwidth is provided to the traffic. The limitless bandwidth can guarantee the transmission of the traffic at its peak rate. An allocation of the peak rate leads to inefficient utilization of network resources. The inefficiency can be improved by adapting a bandwidth renegotiation method according to the invention.

The conditions for 'no delay VBR' transmission comprise $D=0$ and $B_{\max}^e + B_{\max}^d = 0$. The conditions of traffic parameters for the bandwidth renegotiation are preferably as follows.

To transmit the coded data $E_i(Q_{igt})$ with no delay, the upper bound of transmission rate is preferably $R_i^+ = \min\{E_i(Q_{igt}), L_{\max} - L_{i-1} + R_s\}$ from equation (5). Therefore, if $E_i(Q_{igt}) \leq L_{\max} - L_{i-1} + R_s$, $R_i^+ = E_i(Q_{igt})$ for all i . It is noted that $E_i(Q_{igt}) \leq E_{\max}(Q_{igt})$ and $R_s \leq L_{\max} - L_{i-1} + R_s \leq R_p$. If $E_{\max}(Q_{igt}) \leq L_{\max} - L_{i-1} + R_s = R_p$ for $E_i(Q_{igt}) = E_{\max}(Q_{igt})$ and if the average of $L_{\max} - L_{i-1} + R_s$, ($j=i-G_{N+1}, \dots, i$) is greater than $E_{avg}(Q_{igt})$ for any GOP, all data can be transmitted without delay. From equations (15) and (17), a proper condition for 'no delay VBR' are $K_m > 1$, $K_s > 0$, and $K_p = 1$ with the requirements $D=0$ and $B_{\max}^e + B_{\max}^d = 0$. If the negotiated traffic parameters are R_s , R_p , MBL, then the size of LB counter is $R_p - R_s$ from equation (20) and the upper bound of transmission rate is

$R_i^+ = \min\{E_i, L_{\max} - L_{i-1} + R_s\}$ from equation (5). As a result, the encoder rate is preferably controlled so that $E_i \leq R_i^+$.

In the case of 'smoothed VBR', smoothing is a preferred method to protect traffic flows from being lost as well as to improve network utilization. The present invention provides a traffic smoothing method that is capable of reducing a peak rate to a sustainable rate. It is noted that the traffic smoothing requires $D > 0$ and $B_{\max}^e + B_{\max}^d > 0$.

From equations (16) and (17), there are two specific cases that the transmission rate R_i is always less than or equal to the peak rate R_p , and the average of the R_i is less than or equal to the sustainable rate R_s .

When $K_p = 1$, the peak rate R_p is greater than or equal to the sustainable rate R_s . When $K_p = 0$, the upper bound of transmission rate R_i can be determined by $R_p = R_s$. In other words, the transmission rate R_i is preferably smoothed to the sustainable rate R_s where $D > 0$. The

'smoothed VBR' is defined as $K_p = 0$ and $D > 0$. If $K_p < 1$, the maximum bit rate $E_{\max}(Q_{igt})$ is smoothed by equation (17). In the case of 'smoothed VBR', since $L_{\max} = 0$ from equation (20), the upper bound of channel rate of equation (5), i.e. the maximum number of bits capable of being transmitted during i-th frame period R_i^+ is given by

$$R_i^+ = \min\{B_{\max}^d - B_{i-1}^d + E_{i-D}, R_s\} \quad (21)$$

To transmit the coded data $E_i(Q_{igt})$ with Q_{igt} at $R_i(\leq R_s)$, the condition $E_i(Q_{igt}) = E_i^+$ is preferably satisfied. When $B_{i-1}^d = 0$, the maximum number of bits R_i^+ is limited to the sustainable rate R_s from

equation (21). When $R_i^+ = R_s$, ($j=1, i+1, \dots, i+D$), the largest data can be transferred by channel. In this case, the condition

$E_i(Q_{tgt}) = E_i^+ = B_{i+D-1}^d + R_s$ from equation (6) is given by

$$\sum_{j=0}^D E_{i-j}(Q_{tgt}) \leq R_s(D+1) \quad (22)$$

Equations (4) and (22) are conditions selected so that all $E_i(Q_{tgt})$ can be transmitted at the smoothed rate $R_i (= R_s)$. Since the smaller D makes R_s be more closed to $E_{\max}(Q_{tgt})$, the constant K_s should be closed to 1. When the delay D is assumed to be equal to the total length of sequence, all frames are coded with Q_{tgt} and then transmitted to constant bit rate $E_{avg}(Q_{tgt})$.

For example, a MPEG sequence is assumed to be GOP structure of IBPBPBPBPBPB. In case that the delay is D frame, among several successive $(D+1)$ frames satisfying the conditions of equations (4) and (22), when I-frame is included in the $(D+1)$ frames, the maximum number of bits is generated. It is assumed that the GOP structure is '1, 2, 3'. When $D = 3$ and the data ratio of I-frame, P-frame, and B-frame for consistent visual quality is $x:y:z$, the smoothed sustainable rate $R_s = (x+y+2z)U/4$, where U is constant for reflecting the ratio. If $x:y:z = 4:2:1$, $E_{\max}(Q_{tgt})(=x):E_{avg}(Q_{tgt}):R_s = 4:5/3:2$. In this case, the smoothed rate or sustainable rate R_s is half of $E_{\max}(Q_{tgt})$ and 20 % of $E_{avg}(Q_{tgt})$. If $D = 5$, R_s is 10% larger than $E_{avg}(Q_{tgt})$.

According to the present invention, a variable bit rate MPEG

video with consistent quality can be transmitted in real time.

Although illustrative embodiments have been described herein with reference to the accompanying drawings, it is to be understood that the present invention is not limited to those precise embodiments, and
5 that various other changes and modifications may be affected therein by one skilled in the art without departing from the scope and spirit of the invention. It is to be understood that all such changes and modifications are intended to be included within the scope of the invention as defined by the appended claims.

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